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**Remark:** In the Tutorial Session and Office Hours we can discuss

## Probability Theory

**Problem 1 (Random Sets)** *In this problem we consider an alternate way to pick a random subset  $S \subseteq \{1, 2, \dots, n\}$ .*

1. *We generate a subset  $X$  of this set as follows: a fair coin is flipped independently for each element of the set: if the coin lands heads then the element is added to  $X$ , and otherwise it is not. Argue that the set  $X$  is equally likely to be any one of the  $2^n$  possible subsets.*
2. *Suppose that two sets  $X$  and  $Y$  are chosen independently and uniformly at random from all the  $2^n$  subsets of  $\{1, 2, \dots, n\}$ . Determine  $\mathbb{P}(X \subseteq Y)$  and  $\mathbb{P}(X \cup Y = \{1, 2, \dots, n\})$ .*

**Problem 2 (False Positive)** *A medical company has developed a new test for a certain genetic disorder. The false negative rate is small: if you have the disorder, the probability that the test returns a positive result is 0.999. The false positive rate is also small: if you do not have the disorder, the probability that the test returns a positive result is only 0.005. Assume that 2% of the population has the disorder. If a person chosen uniformly from the population is tested and the result comes back positive, what is the probability that the person has the disorder?*